

# AN APPLICATION OF CONDITIONAL COPULA FOR THE RISK MANAGEMENT OF FIXED INCOME PORTFOLIOS

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## ABSTRACT

*Empirical studies show that asset returns are non-normal and also time varying. These findings explain why the normality VaR and unconditional VaR perform poorly in market risk measurement and management. In the literature, the bivariate conditional copula function can be used to represent a joint distribution function of correlated asset returns whose marginal distributions can be of any form, and to accommodate time varying behavior of asset returns. This study extends the technique further to be able to measure the risk of a portfolio of any number of assets. The extension is successful because the study can endorse a positive definite correlation matrix of the conditional copula function by applying Semidefinite Programming (SDP). The study applies the method in risk measurement of Thai Government bond portfolios. In principle, the method should show improved performance over previous empirical studies, but the results from the Kupiec test show that the conditional copula method fails to estimate the VaR of these portfolios. This failure may have been caused by the misspecification of marginal distribution.*

**Keywords:** Value at Risk; Conditional Copula; Risk Measurement

## 1. INTRODUCTION

Nowadays, risk management plays an important role in the asset management industry. Among available techniques for risk measurement and management, Value at Risk and Expected Shortfall are the most popular (Artzner et al., 1997). To apply the VaR or ES techniques, users must define a joint distribution of asset returns, which principally is a multi-normal distribution (J.P. Morgan and Reuters, 1996). In Thailand, there is evidence that the characteristics of returns of fixed income securities usually differ from a joint-normal distribution (Khanthavit, 2006). If returns are not normal, the standard methods are inadequate. Furthermore, analytical formulae and mathematical models for risk measurement and management based on a non-normality distribution assumption are either difficult or nonexistent (Embrechts et al., 2002). As a result, an alternative to solving this problem is to use the simulation and numerical approach. For simulation, users need to generate a distribution of asset returns whose marginal distributions are those of particular assets in the portfolio. Moreover, the joint distribution must specifically describe the covariation among these returns. When the marginal distributions of the returns are of different types, the form of the joint distribution is usually unknown, hence constituting a more difficult problem to solve in a direct way. Therefore, the objective of this study is to find a method for modeling joint distributions when marginal distributions of asset returns are different and when returns covary.

The study proposes a copula approach. For this particular problem, copula can be useful because its transformed function can represent a joint distribution function of correlated asset returns whose marginal distributions can be of any form (Nelson, 1999; Sklar, 1973). The use of copula for risk measurement and management problems is not new. In the past, many studies provided technical notes as to how copula could be applied (Di Clemente and Romano, 2003; Embrechts et al., 2003; Khanthavit, 2006; Micocci and Masala, 2004; Romano, 2002). Most empirical results pointed to the success of the copula technique (Di Clemente and Romano, 2003; Micocci and Masala, 2004; Romano, 2002). The resulting VaR and ES are more reliable in backtesting and stress testing. Despite its success in general, Khanthavit (2006) found that the copula VaR of fixed income portfolios in Thailand underestimates and is less accurate than the normality VaR. The problem may be that the statistical characteristics of returns of Thai Government Bond Indices change over time (Tasakasiri, 2004), which in turn makes the copula method, which